Nucleon Mass Corrections to Spin Dependent Structure Functions and Relations Between their Twist-3 Contributions

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The nucleon mass corrections are calculated to all polarized structure functions for neutral and charged current deep inelastic scattering in lowest order in the coupling constant. The impact of the target mass corrections on the general relations between the twist–2 and twist–3 parts of the structure functions is studied and three new relations between the twist–3 contributions are derived. The size of nucleon mass corrections for the g_1 and g_2 structure functions are estimated.

1. INTRODUCTION

In the experiments in which polarized lepton scattering off polarized targets has been studied so far the most data are taken in the range of lower values of Q^2. In this domain nucleon–mass corrections have to be taken into account to analyze the Q^2 dependence of structure functions (SF). The target mass effects of $O((M^2/Q^2)^k)$ form one contribution to the power corrections. Unlike the case for dynamical higher twist effects the target mass corrections can be calculated in closed form in all orders in M^2/Q^2 for deep inelastic SF.

Little is known so far on the relative strength of dynamical higher twist operators and their scaling violations. Before one may try to pursue a common treatment various conceptional problems concerning higher twist operators have first to be solved. Due to this we limit the present analysis to a systematic study of the target mass corrections for all deep inelastic structure functions extending earlier investigations [1, 2, 3, 4]. From an experimental point of view, on the other hand, the kinematic higher twist effects emerge as a background for the extraction of the dynamical higher twist terms at moderate energies.

Another goal of our investigation is to study the effect of the target mass corrections on the integral relations between different polarized structure functions in lowest order in the coupling constant. As in Ref. [5] nucleon mass effects were disregarded, except of those implied by the kinematics in the Born cross sections, twist–3 contributions to the structure functions g_1, g_4 and g_5 were not yet obtained. This picture has to be regarded as partly incomplete, since nucleon mass effects have either to be accounted for thoroughly or to be neglected at all. In the latter case, however, the scattering cross sections for longitudinally polarized nucleons would not even contain the structure functions g_2 and g_3.

Two methods were proposed in the literature for the evaluation of the target mass corrections. Nachtmann [6] translated the usual power–series expansion into an expansion of operators of definite spin. The problem may be solved by applying the representation–theory of the Lorentz group. A second method, which we used in our calculations, was proposed by Georgi and Politzer [1] and relies on resummation of the individual mass terms.

2. RESUMMED EXPRESSIONS FOR THE STRUCTURE FUNCTIONS

We consider the light-cone expansion of the forward Compton scattering amplitude (FCSA) in momentum space. As a result, the T-product of two charged currents can be expressed through one quarkonic operator at leading order QCD, $\Theta^{\pm\beta\{\mu_1,\cdots\mu_n\}} = \bar{q}\gamma_5\gamma_\beta D_{\mu_1}\cdots D_{\mu_n}q$. The sign $\pm$ in the Θ-operators corresponds to the charged

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currents combinations

$$T^{\pm}_{\mu\nu}(q^2,\nu) = T^{W^-}_{\mu\nu}(q^2,\nu) \pm T^{W^+}_{\mu\nu}(q^2,\nu) \ . \qquad (1)$$

For neutral currents all expressions are the same as for $T^{+}_{\mu\nu}$. Decomposing the $\Theta^{\pm}$ operators into a symmetric part and a remainder one can isolate the twist-2 and twist-3 contribution in the FCSA. To find the target mass dependence of the spin dependent structure functions we have to construct the traceless nucleon matrix elements of above operators using two vectors, the nucleon momentum P and the spin vector S. The Θ-operators are traceless in the massless quark limit. As an example, we present here the expression for such a tensor for the twist-2 part of the quark operator

$$\langle PS|\Theta_S^{\pm\mu_1\{\mu_2\cdots\mu_n\}}|PS\rangle =$$
$$a^{\pm}_{n-1}\frac{1}{n}\sum_{j=0}\frac{(-1)^j}{2^j}\frac{(n-j)!}{n!}\underbrace{g\cdots g}_{j}\ \underbrace{[SP\cdots P]_S}_{n-2j}M^{2j}.$$

The sum contains j times the metric tensors $g_{\mu_i\mu_k}$. The remaining $n-2j$ indices are symmetrized in the product $[SP\ldots P]_S$; $a^{\pm}_n$ denotes the twist-2 operator reduced matrix elements. The same expressions can be written for the twist-3 part of the Θ-operator using the reduced matrix elements $d^{\pm}_n$. These are non-perturbative quantities and are independent of the nucleon mass. All information about the target mass corrections is contained in the tensor structures of the nucleon matrix elements. Taking the nucleon matrix elements of the T-product of two currents one obtains the FCSA. The dispersion relations result into expressions of the moments of structure functions which are expressed through series of the ratio $(M^2/Q^2)^k$ (see details in [7]). In many practical applications, as the analysis of experimental data, the expressions for the moments of the deep inelastic SF are less suited than the corresponding x-space expression. To obtain these we apply we apply the inverse Mellin transform to the moments of the corresponding SF. For the twist-2 parts of the polarized SF we have

$$g_1^{\pm\ tw2}(x) = x\frac{d}{dx}x\frac{d}{dx}\left[\frac{x}{y}\frac{G_1^{\pm}(\xi)}{\xi}\right], \qquad (2)$$

$$g_2^{\pm\ tw2}(x) = -x\frac{d^2}{dx^2}x\left[\frac{x}{y}\frac{G_1^{\pm}(\xi)}{\xi}\right], \qquad (3)$$

$$g_3^{\pm\ tw2}(x) = 2x^2\frac{d^2}{dx^2}\left[\frac{x^2}{y}\frac{G_2^{\pm}(\xi)}{\xi^2}\right], \qquad (4)$$

$$g_4^{\pm\ tw2}(x) = -x^2\frac{d}{dx}x\frac{d^2}{dx^2}\left[\frac{x^2}{y}\frac{G_2^{\pm}(\xi)}{\xi^2}\right], \qquad (5)$$

$$g_5^{\pm\ tw2}(x) = -x\frac{d}{dx}\left[\frac{x}{y}\frac{G_3^{\pm}(\xi)}{\xi}\right] + \frac{M^2}{Q^2}x^2\frac{d^2}{dx^2}\left[\frac{x^2}{y}\frac{G_2^{\pm}(\xi)}{\xi}\right], \qquad (6)$$

where $y=\sqrt{1+4M^2x^2/Q^2}$ and ξ is the Nachtmann variable, $\xi = 2x/\left[1+(1+4M^2x^2/Q^2)^{1/2}\right]$ [6]. The operator expectation values $a^{\pm}_n$ are the moments of distribution functions $F^{\pm q}(x)$, which are related to the polarized parton densities in the massless limit, $\Delta q(x)\pm\Delta\overline{q}(x)$,

$$a_n^{\pm,q} = \int_0^1 dy y^n F^{\pm q}(y) \ . \qquad (7)$$

The functions $G^{\pm}_i$ are related to the distribution function $F^{\pm q}(y)$ by

$$G_1^{\pm}(y) = \sum_q \frac{(g_V^q)^2+(g_A^q)^2}{4}\int_y^1\frac{dy_1}{y_1}\int_{y_1}^1\frac{dy_2}{y_2}F^{\pm q}(y_2) \ ,$$

$$G_2^{\pm}(y) = \sum_q g_V^q g_A^q\int_y^1 dy_1\int_{y_1}^1\frac{dy_2}{y_2}\int_{y_2}^1\frac{dy_3}{y_3}F^{\pm q}(y_3) \ ,$$

$$G_3^{\pm}(y) = \sum_q\frac{1}{2}\ g_V^q g_A^q\int_y^1\frac{dy_1}{y_1}F^{\pm q}(y_1) \ .$$

In the same way the moments of the corresponding twist-3 contributions can be inverted. One obtains the following x-space expressions:

$$g_1^{\pm\ tw3}(x,Q^2) = \frac{4M^2}{Q^2}x^2\frac{d^2}{dx^2}\left[\frac{x^2}{y}H_1^{\pm}(\xi)\right], \qquad (8)$$

$$g_2^{\pm\ tw3}(x,Q^2) = x\frac{d^2}{dx^2}\left[\frac{x}{y}H_1^{\pm}(\xi)\right], \qquad (9)$$

$$g_3^{\pm\ tw3}(x,Q^2) = -\left(x^2\frac{d^3}{dx^3}+4\frac{M^2x^2}{Q^2}x\frac{d^2}{dx^2}x\right)\times\left[\frac{x^2}{y}\frac{H_2^{\pm}(\xi)}{\xi}\right], \qquad (10)$$

$$g_4^{\pm\ tw3}(x,Q^2) = -4\frac{M^2}{Q^2}x^3\frac{d^3}{dx^3}\left[\frac{x^3}{y}\frac{H_2^{\pm}(\xi)}{\xi}\right], \qquad (11)$$

$$g_5^{\pm\ tw3}(x,Q^2) = \quad -2\frac{M^2}{Q^2}x^2\frac{d^2}{dx^2}\left[\frac{x^2}{y}\frac{H_2^{\pm}(\xi)}{\xi}\right].(12)$$

The matrix elements of the twist–3 operators are the moments of the distribution function $D^{\pm q}(x)$ in the massless limit,

$$d_n^{\pm q} = \int_0^1 dx x^n D^{\pm q}(x) \ , \qquad (13)$$

which has, however, no partonic interpretation.

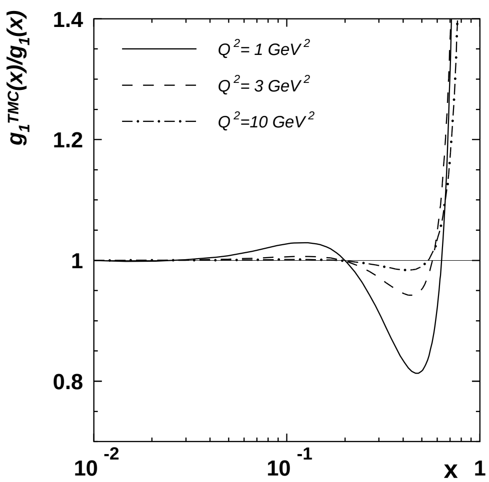

Figure 1. The ratio $g_1^{TMC}(x)/g_1(x)$ versus x.

The functions $H_{1,2}^{\pm}(x)$ can be expressed through integrals over the distribution function $D^{\pm q}(x)$ as in the twist-2 case [7]. Performing the derivatives in Eqs.(2)-(4) and (8)-(12) we obtain the final expressions for the structure functions in x-space. The twist-2 and twist-3 parts are expressed through the $F^{\pm q}$ and $D^{\pm q}$, respectively. The corresponding expressions are given in Ref. [7]. Here we present numerical results on the size of target mass effects for the twist-2 contributions to g_1 and g_2. We calculate $g_1(x)$ using the LO parametrization for polarized SF [10]. To estimate the size of nucleon mass corrections for the twist-2 part of $g_1(x)$ we use the same expression for $g_1(x)$ as $F^q(x)$. The ratio of $g_1^{TMC}(x)/g_1(x)$ is shown in Fig. 1 for different values of Q^2. The same calculation is done for the structure function $g_2(x)$, Fig. 2. Here $g_2(x)$ was calculated

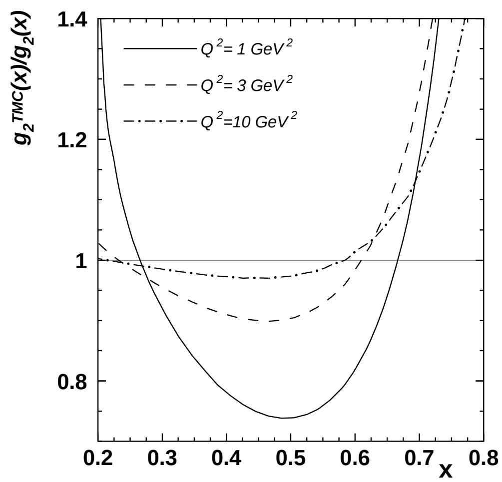

Figure 2. The ratio $g_2^{TMC}(x)/g_2(x)$ versus x.

from $g_1(x)$ using the Wandzura-Wilczek relation [8].

3. RELATIONS BETWEEN THE STRUCTURE FUNCTIONS

As was shown in a previous analysis [5] the twist-2 contributions to the polarized structure functions are connected by three (integral) relations, the Dicus–relation [9], the Wandzura–Wilczek (WW) relation [8] and a new relation by Blümlein and Kochelev [5]. It can easily be seen from Eqs. (2)-(6) that the nucleon mass correction does not affect the WW relation as well as the relation of Ref. [5]. Moreover, the WW relation is not violated even by quark mass corrections [7]. The Dicus relation is violated in the presence of nucleon mass corrections similarly to the Callan–Gross relation [11] in the unpolarized case.

In the presence of target mass corrections all structure functions $g_i|_{i=1}^5$ contain twist–3 contributions. On the contrary, in the massless limit this is the case for the structure functions g_2 and g_3 only [5]. The mass dependence of the scattering cross sections for longitudinal nucleon polarization, however, reveals that both the structure functions g_2 and g_3 do only contribute at $O(M^2/Q^2)$. Therefore, a complete account for

twist–3 contributions requires to consider also the nucleon mass corrections for the other polarized structure functions. From Eqs. (8–12) one derives the following *new* relations between the twist–3 parts of the different spin–dependent structure functions :

$$g_1^i(x,Q^2) = \frac{4M^2x^2}{Q^2}\left[g_2^i(x,Q^2) - 2\int_x^1 \frac{dy}{y} g_2^i(y,Q^2)\right], \quad (14)$$

$$\frac{4M^2x^2}{Q^2} g_3^i(x,Q^2) = g_4^i(x,Q^2)\left(1 + \frac{4M^2x^2}{Q^2}\right) + 3\int_x^1 \frac{dy}{y} g_4^i(y,Q^2), \quad (15)$$

$$2xg_5^i(x,Q^2) = -\int_x^1 \frac{dy}{y} g_4^i(y,Q^2). \quad (16)$$

Here, Eq. (14) at the one side, and Eqs. (15,16) on the other side correspond to different flavor combinations among the twist–3 contributions. This is similar to the case of the twist–2 terms, where the former case corresponds to the combination $\Delta q + \Delta\overline{q}$, and the latter to $\Delta q - \Delta\overline{q}$.

Eqs. (14–16) show that the twist–3 contributions to g_1, g_4 and g_5 vanish in the limit $M \rightarrow 0$. On the other hand, if one keeps terms of $O\left[(M^2/Q^2) \cdot g_{2(3)}\right]$ and twist–3 in the scattering cross sections, one has to account also for the twist–3 terms in g_1, g_4 and g_5.

4. CONCLUSION

We have calculated the target mass corrections for all polarized structure functions for both neutral and charged current deep inelastic scattering. The results were obtained by using the local light cone expansion of the FCSA. The target mass corrections imply besides the twist–2 terms twist–3 contributions for all polarized structure functions. The corrections were both represented in terms of the integer moments which result from the light cone expansion and their analytic continuation and Mellin inversion to x-space. The size of nucleon mass corrections is expected to be $20\% - 40\%$ for $Q^2 \simeq 1$ GeV^2 at moderate values of x for the twist-2 contributions of $g_1(x)$ and $g_2(x)$.

We investigated the effect of the target mass corrections on the sum rules connecting the polarized structure functions in lowest order in the coupling constant. For the twist–2 contributions both the Wandzura–Wilczek relation [8] and the relation derived in Ref. [5] are preserved, whereas the Dicus relation [9] receives a correction. It was also shown that the Wandzura–Wilczek relation is preserved in the presence of quark–mass corrections.

Three new integral relations were derived for the twist–3 contributions of the polarized structure functions. They hold without further assumptions on the flavor combinations of the related structure functions.

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